TEM EQUIVALENT CIRCUITS FOR QUASI-TEM COUPLERS

Sheng Cheng and Marion Lee Edwards, Ph.D.

The Johns Hopkins University
Whiting School of Engineering and Applied Physics Laboratory

ABSTRACT

Equivalent circuits for TEM couplers (1,2), important in the design of coupled line filters, E.G. interdigital or combline, are inadequate approximations for quasi-TEM structures such as microstrip. Exact quasi-TEM coupler equivalent circuits are developed consisting of TEM line configurations suitable for CAE design and analysis applications.

INTRODUCTION

The voltage and current vectors for a coupled line are related by,

$$\frac{\partial \mathbf{V}}{\partial z} = -j\omega\, \mathbb{L}\, \mathbf{I}, \qquad \frac{\partial \mathbf{I}}{\partial z} = -j\omega\, \mathbb{C}\, \mathbf{V}$$

where

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix},$$

$$\mathbb{L} = \begin{bmatrix} L_1 & L_m \\ L_m & L_2 \end{bmatrix} \qquad \mathbb{C} = \begin{bmatrix} C_1+C_m & -C_m \\ -C_m & C_2+C_m \end{bmatrix}$$

These result in the second order equations

$$\frac{\partial^2 \mathbf{V}}{\partial z^2} + \omega^2\, \mathbb{L}\, \mathbb{C}\, \mathbf{V} = \mathbb{0},$$

$$\frac{\partial^2 \mathbf{I}}{\partial z^2} + \omega^2\, \mathbb{C}\, \mathbb{L}\, \mathbf{I} = \mathbb{0}$$

where

$$\mathbb{L}\,\mathbb{C} = \begin{bmatrix} L_1C_1+L_1C_m-L_mC_m & L_mC_2+L_mC_m-L_1C_m \\ L_mC_1+L_mC_m-L_2C_m & L_2C_2+L_2C_m-L_mC_m \end{bmatrix}$$

$$= (\mathbb{C}\,\mathbb{L})^T$$

Since a TEM line has a unique propagation velocity the matrix $\mathbb{LC}$ must be a diagonalized, i.e.,

$$\mathbb{LC} = \mathbb{CL} = \mathcal{V}_p^2 \cdot \mathbb{I},$$

where $\mathcal{V}_p$ is the phase velocity of the medium and $\mathbb{I}$ is the identity matrix. This imposes the well known relationship between the inductances and capacitances,

$$L_{1,2} = \frac{C_{2,1} + C_m}{\mathcal{V}_p^2\ (C_1C_2 + C_1C_m + C_2C_m)},$$

and

$$L_m = L_{1,2} \frac{C_m}{C_{2,1} + C_m}$$

Since capacitances and phase velocity are sufficient to represent propagation on TEM line we see that a TEM coupler must be equivalent to a series configuration of three transmission lines (2) shown in Figure 1. This equivalent circuit

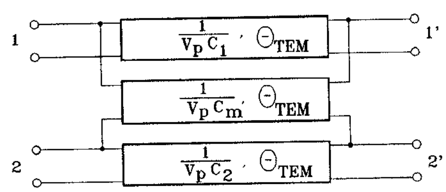

Figure 1. The three line equivalent circuit for a TEM coupler.

has greatly facilitated coupled line analysis critical to the design of interdigital and combline filters. A similar equivalent circuit for microstrip couplers must account for the quasi-TEM nature of the medium.

QUASI-TEM SYMMETRIC COUPLERS

For a symmetric quasi-TEM coupled line the matrix product,

$$\mathbb{L}\,\mathbb{C} = \begin{bmatrix} LC+LC_m-L_mC_m & L_mC+L_mC_m-LC_m \\ L_mC+L_mC_m-LC_m & LC+LC_m-L_mC_m \end{bmatrix}$$

unlike a TEM coupled line, is not a diagonalized matrix. However, the linear transformation,

$$\mathbb{T} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

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defines the *even* and *odd* modes, $\tilde{\mathbf{V}}$ and $\tilde{\mathbf{I}}$ by

$$\mathbf{V} = \mathbb{T}\,\tilde{\mathbf{V}} \quad \text{and} \quad \mathbf{I} = \mathbb{T}\,\tilde{\mathbf{I}},$$

results in the second order equation for the orthogonal voltage mode

$$\frac{\partial^2 \tilde{\mathbf{V}}}{\partial z^2} + \omega^2\, \mathbb{T}^{-1}\, \mathbb{L}\, \mathbb{C}\, \mathbb{T}\, \tilde{\mathbf{V}} = \mathbb{O}$$

The matrix product

$$\mathbb{T}^{-1}\, \mathbb{L}\, \mathbb{C}\, \mathbb{T} = \begin{bmatrix} (L+L_m)C & 0 \\ 0 & (L-L_m)(C+2C_m) \end{bmatrix}$$

$$= \begin{bmatrix} \mathcal{V}_{even}^2 & 0 \\ 0 & \mathcal{V}_{odd}^2 \end{bmatrix}$$

determines the wave propagation velocity which depends upon the modes.

The even and odd mode capacitances are found from

$$\mathbb{T}^{-1}\, \mathbb{C}\, \mathbb{T} = \begin{bmatrix} C & 0 \\ 0 & (C+2C_m) \end{bmatrix} = \begin{bmatrix} C_{even} & 0 \\ 0 & C_{odd} \end{bmatrix}$$

and the even and odd mode characteristic impedances are

$$Z_{even} = [\mathcal{V}_{even}\, C\,]^{-1}$$

$$Z_{odd} = [\mathcal{V}_{odd}\, (C+2C_m)\,]^{-1}$$

Since each of the orthogonal modes is by itself a TEM system, then it is possible to utilize the three-line equivalent circuit of the TEM coupled line to construct a similar type equivalent circuit for the quasi-TEM coupled line. This is carried out by reversing the transformation, i.e.,

$$\tilde{\mathbf{V}} = \mathbb{T}^{-1}\, \mathbf{V}\ ; \quad \tilde{\mathbf{I}} = \mathbb{T}^{-1}\, \mathbf{I},$$

and separating the capacitance contribution for each mode. Consequently,

$$\frac{\partial \mathbf{I}}{\partial z} = -j\omega\, \mathbb{T} \begin{bmatrix} C_{even} & 0 \\ 0 & C_{odd} \end{bmatrix} \mathbb{T}^{-1}\, \mathbf{V}$$

in which

$$\mathbb{T} \begin{bmatrix} C_{even} & 0 \\ 0 & C_{odd} \end{bmatrix} \mathbb{T}^{-1} =$$

$$(1/2) \begin{bmatrix} C_{even} & C_{even} \\ C_{even} & C_{even} \end{bmatrix} + (1/2) \begin{bmatrix} C_{odd} & -C_{odd} \\ -C_{odd} & C_{odd} \end{bmatrix}$$

The capacitance matrix for the quasi-TEM line is the sum of two matrices each of which represent TEM lines. The two TEM capacitive networks are shown in Figure 2, and can be replaced by the three-line equivalent circuit. In the first matrix one of the lines has a negative impedance. In the second matrix two of the lines to ground are seen to be of infinite impedance and can therefore be omitted resulting in an equivalent circuit consisting of one line. The total equivalent circuit, therefore, consists of four TEM lines of two different lengths equating to the even and odd phase velocities.

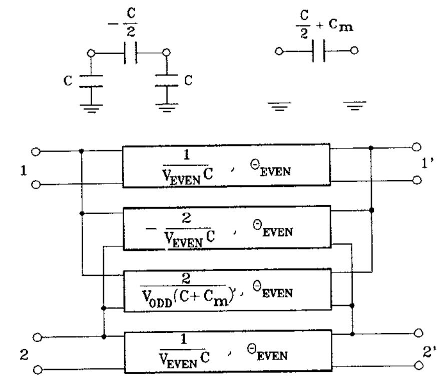

Figure 2. The four line equivalent circuit resulting from the four capacitances for a symmetric quasi-TEM coupler

An understanding of a line with a negative characteristic impedance is obtained by looking at its S-parameters, i.e.,

$$S_{11}(-Z_c) = \frac{-j(Z_c^2 - Z_o^2)\,\mathrm{Tan}\,\theta}{2Z_cZ_o - j(Z_c^2+Z_o^2)\,\mathrm{Tan}\,\theta} = S_{11}^*(Z_c)$$

$$S_{21}(-Z_c) = \frac{Z_o\,[1+S_{11}(-Z_c)]}{Z_o\mathrm{Cos}\,\theta - jZ_c\,\mathrm{Sin}\,\theta} = S_{21}^*(Z_c)$$

A negative characteristic impedance line has S-parameters which are the complex conjugate of the positive line. Thus, the loci of S_{11} and S_{21} on the Smith Chart move in the opposite circular direction from those of S_{11} and S_{21}. Alternatively, a $-Z_c$ transmission line with length θ acts like a Z_c line with length $-\theta$.

QUASI-TEM ASYMMETRIC COUPLER

Since the matrix products $\mathbb{L}\,\mathbb{C}$ and $\mathbb{C}\,\mathbb{L}$ do not result in symmetric matrices then two different transformations, namely $\mathbb{T}_V$ and $\mathbb{T}_I$, are needed to diagonalize them, respectively. However, since $\mathbb{L}\,\mathbb{C}$ and $\mathbb{C}\,\mathbb{L}$ have the same set of eigenvalues, the orthogonal mode propagation velocities for the voltage and current waves are the same. This is expressed as

$$\begin{bmatrix} 1/\mathcal{V}_c^2 & 0 \\ 0 & 1/\mathcal{V}_\pi^2 \end{bmatrix} = \mathbb{T}_V^{-1}\, \mathbb{L}\, \mathbb{C}\, \mathbb{T}_V = \mathbb{T}_I^{-1}\, \mathbb{C}\, \mathbb{L}\, \mathbb{T}_I$$

where

$$\mathbb{T}_V = \begin{bmatrix} 1 & 1 \\ R_{Vc} & R_{V\pi} \end{bmatrix} \text{ and } \mathbf{V} = \mathbb{T}_V\, \tilde{\mathbf{V}}$$

$$\mathbb{T}_I = \begin{bmatrix} 1 & 1 \\ R_{Ic} & R_{I\pi} \end{bmatrix} \text{ and } \mathbf{I} = \mathbb{T}_I\, \tilde{\mathbf{I}}$$

$\mathcal{V}_c$ and $\mathcal{V}_\pi$ are wave propagation velocities for the c and π orthogonal modes, respectively, where c denotes the *co-phase* or generalized even mode and π denotes the *anti-phase (π radians)* or generalized odd mode.

If the $\mathbb{L}\mathbb{C}$ matrix satisfies the congruence condition (3,4),

$$\mathbb{L}\,\mathbb{C} = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

where

$$A + B = C + D$$

then

$$C_1 \,/\, C_2 = (L_1 - L_m) \,/\, (L_2 - L_m).$$

This equation expresses the physical meaning described by Speciale: "... the congruence condition may be expressed by saying that the introduction of the nonhomogeneous dielectric in the empty structure must not affect the value of the ratio of the per-unit-length line-to-ground capacitances."

When the congruence condition is met, the propagation velocities of the orthogonal modes and the transformation matrices are simplified into the following forms.

$$\mathcal{V}c = \frac{1}{\sqrt{L_1C_1 + L_mC_2}} = \frac{1}{\sqrt{L_2C_2 + L_mC_1}}$$

$$\mathcal{V}\pi = \frac{1}{\sqrt{(L_1-L_m)(C_1-C_m) + (L_2-L_m)C_m}} = \frac{1}{\sqrt{(L_2-L_m)(C_2-C_m) + (L_1-L_m)C_m}}$$

and

$$\mathbb{T}v = \begin{bmatrix} 1 & 1 \\ 1 & -C_1/C_2 \end{bmatrix}$$

$$\mathbb{T}_I = \begin{bmatrix} 1 & 1 \\ C_2/C_1 & -1 \end{bmatrix}$$

The orthogonal mode capacitances are given by

$$\mathbb{T}_I^{-1}\,\mathbb{C}\,\mathbb{T}v = \begin{bmatrix} C_1 & 0 \\ 0 & C_1+[(C_1+C_2)/C_2]C_m \end{bmatrix}$$

$$= \begin{bmatrix} Cc & 0 \\ 0 & C\pi \end{bmatrix}$$

Again the TEM line equivalent circuit is found by reversing the transformation to get

$$\mathbb{C} = \mathbb{T}_I \begin{bmatrix} Cc & 0 \\ 0 & C\pi \end{bmatrix} \mathbb{T}v^{-1}$$

or

$$\mathbb{C} = C_2/(C_1+C_2) \begin{bmatrix} (C_1/C_2)Cc & Cc \\ Cc & (C_2/C_1)Cc \end{bmatrix}$$

$$+ \; C_2/(C_1+C_2) \begin{bmatrix} C\pi & -C\pi \\ -C\pi & C\pi \end{bmatrix}$$

In the above equation, c and π mode are separated in the original $\mathbb{C}$ matrix as the superposition of two matrices. We can thus represent each matrix with a capacitive network. And since each network now has only one propagation velocity, it is possible to replace each capacitor in the network with a TEM transmission line to construct the complete equivalent circuit.

Substitution of the expressions for Cc and $C\pi$ into the above expression results in

$$\mathbb{C} = \begin{bmatrix} C_1^2/(C_1+C_2) & C_1C_2/(C_1+C_2) \\ C_1C_2/(C_1+C_2) & C_2^2/(C_1+C_2) \end{bmatrix} +$$

$$\begin{bmatrix} [C_1C_2/(C_1+C_2)]+C_m & -[C_1C_2/(C_1+C_2)] - C_m \\ -[C_1C_2/(C_1+C_2)] - C_m & [C_1C_2/(C_1+C_2)]+C_m \end{bmatrix}$$

The corresponding TEM networks are shown in Figure 3. It can be easily verified that by setting $C_1 = C_2$, the above equations reduce to exactly those derived for the symmetric quasi-TEM coupled line.

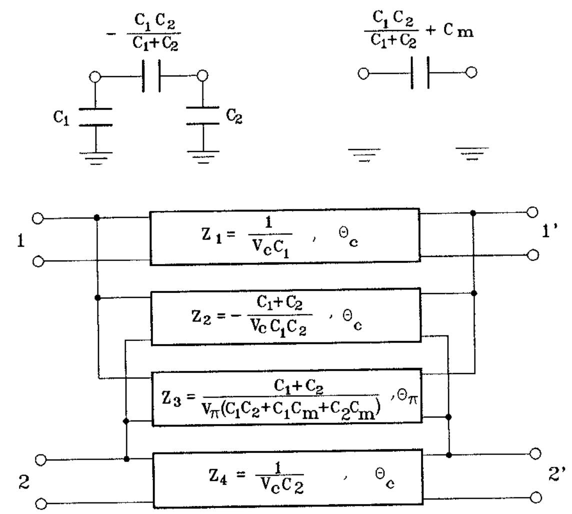

Figure 3. The four line equivalent circuit for an asymmetric quasi-TEM coupler, assuming $\mathbb{L}\mathbb{C}$ is congruent.

Quasi-TEM Parallel Line Two Port Circuits

Various papers presented equivalent circuits for assorted Quasi-TEM parallel coupled line two-port prototypes. Among them, there are Zysman and Johnson (6) on symmetric coupled line, Allen (7) on congruent asymmetric coupled line, and Tripathi (8) on general asymmetric coupled line. With the use of the four line equivalent circuit, it is possible to derive alternative equivalent two-port circuits in the previously mentioned paper.

In the two-port equivalent circuit, Richards' Transform is used to replace shorted or opened stub with its equivalent inductor or capacitor respectively. In the case of a negative impedance line, the inductance or capacitance is negative. Using the four line approach Figure 4 shows circuits equivalent for several quasi-TEM coupled line two port elements.

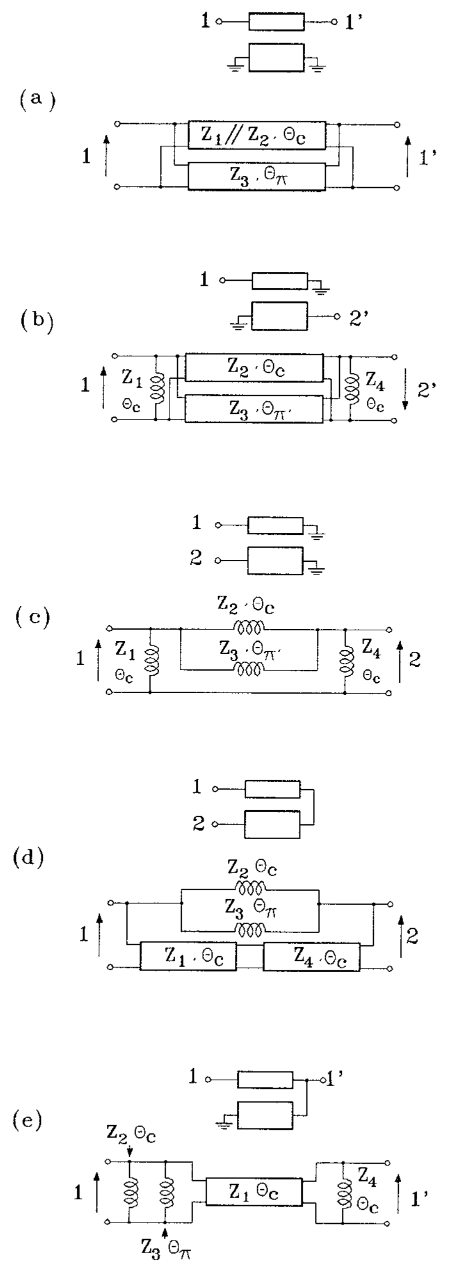

Figure 4. Equivalent circuits for (a) a Shorted Symmetric section, (b) a shorted Interdigital section, (c) a shorted Comb section, (d) a Meander section, and (e) a Spurline section. Note that Z_i's defined in figure 3.

CONCLUSION

Equivalent circuits for quasi-TEM coupled lines have been developed for symmetric line ($C_1 = C_2$) and for asymmetric lines having a congruent LC matrix. It has been shown (5) that the congruence assumption is very general. The error is insignificant provided the line widths is narrower than the substrate thickness, a condition generally met in practical circuits. These equivalent circuits are easy to implement CAE system such as TOUCHSTONE™ which accepts and correctly calculates ideal transmission line performance even for negative characteristic impedances.

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